

Study of $\Lambda_b \rightarrow \Lambda(\phi, \eta^{(\prime)})$ and $\Lambda_b \rightarrow \Lambda K^+ K^-$ decays

C. Q. Geng^{1,2,3}, Y. K. Hsiao^{1,2,3}, Yu-Heng Lin³, and Yao Yu¹

¹*Chongqing University of Posts & Telecommunications, Chongqing, 400065, China*

²*Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan 300*

³*Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300*

(Dated: July 12, 2016)

Abstract

We study the charmless two-body $\Lambda_b \rightarrow \Lambda(\phi, \eta^{(\prime)})$ and three-body $\Lambda_b \rightarrow \Lambda K^+ K^-$ decays. We obtain $\mathcal{B}(\Lambda_b \rightarrow \Lambda\phi) = (3.53 \pm 0.24) \times 10^{-6}$ to agree with the recent LHCb measurement. However, we find that $\mathcal{B}(\Lambda_b \rightarrow \Lambda(\phi \rightarrow) K^+ K^-) = (1.71 \pm 0.12) \times 10^{-6}$ is unable to explain the LHCb observation of $\mathcal{B}(\Lambda_b \rightarrow \Lambda K^+ K^-) = (15.9 \pm 1.2 \pm 1.2 \pm 2.0) \times 10^{-6}$, which implies the possibility for other contributions, such as that from the resonant $\Lambda_b \rightarrow K^- N^*$, $N^* \rightarrow \Lambda K^+$ decay with N^* as a higher-wave baryon state. For $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$, we show that $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta, \Lambda\eta') = (1.47 \pm 0.35, 1.83 \pm 0.58) \times 10^{-6}$, which are consistent with the current data of $(9.3_{-5.3}^{+7.3}, < 3.1) \times 10^{-6}$, respectively. Our results also support the relation of $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta) \simeq \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta')$, given by the previous study.

I. INTRODUCTION

The charmless two-body Λ_b decays of $\Lambda_b \rightarrow pK^-$ and $\Lambda_b \rightarrow p\pi^-$ have been observed by the CDF Collaboration [1] with the branching ratios of $O(10^{-6})$, which are in accordance with the recent measurements on $\Lambda_b \rightarrow \Lambda\phi$ and $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$ by the LHCb Collaboration, given by [2, 3]

$$\begin{aligned}\mathcal{B}(\Lambda_b \rightarrow \Lambda\phi) &= (5.18 \pm 1.04 \pm 0.35_{-0.62}^{+0.67}) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta) &= (9.3_{-5.3}^{+7.3}) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta') &< 3.1 \times 10^{-6}, \text{ (90\% C.L.)}\end{aligned}\tag{1}$$

where the evidence is seen for the η mode at the level of 3σ -significance.

Theoretically, $\Lambda_b \rightarrow p(K^{*-}, \pi^-, \rho^-)$ decays via $b \rightarrow u\bar{u}(d, s)$ at the quark level have been studied in the literature [4–9]. In particular, it is interesting to point out that the direct CP violating asymmetry in $\Lambda_b \rightarrow pK^{*-}$ is predicted to be as large as 20%, which is promising to be observed in the future measurements. On the other hand, the decay of $\Lambda_b \rightarrow \Lambda\phi$ via $b \rightarrow s\bar{s}s$ has not been well explored even though both the decay branching ratio and T-odd triple-product asymmetries [10–12] have been examined by the experiment at LHCb [2]. According to the newly measured three-body $\Lambda_b \rightarrow \Lambda K^+ K^-$ decay by the LHCb Collaboration, given by [13]

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda K^+ K^-) = (15.9 \pm 1.2 \pm 1.2 \pm 2.0) \times 10^{-6},\tag{2}$$

it implies a resonant $\Lambda_b \rightarrow \Lambda\phi, \phi \rightarrow K^+ K^-$ contribution with the signal seen at the low range of $m^2(K^+ K^-)$ from the Dalitz plot. However, to estimate this resonant contribution, one has to understand $\mathcal{B}(\Lambda_b \rightarrow \Lambda\phi)$ in Eq. (1) first. Such a study is also important for further examinations of the triple-product asymmetries [14]. For $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$, the relation of $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta) \simeq \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta')$ found in Ref. [15] seems not to be consistent with the data in Eq. (1). Moreover, the first works on $\Lambda_b \rightarrow \Lambda\eta'$ with the branching ratios predicted to be $O(10^{-6} - 10^{-5})$ in comparison with the data in Eq. (1) were done before the observations of $\Lambda_b \rightarrow p(K^-, \pi^-)$, which can be used to extract the $\Lambda_b \rightarrow \mathbf{B}_n$ transition form factors from QCD models [8, 16]. For a reconciliation, we would like to reanalyze $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$.

In this work, we will use the factorization approach for the theoretical calculations of $\Lambda_b \rightarrow \Lambda\phi$ and $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$ as those in the $\Lambda_b \rightarrow p(K^{*-}, \pi^-, \rho^-)$ decays [8].

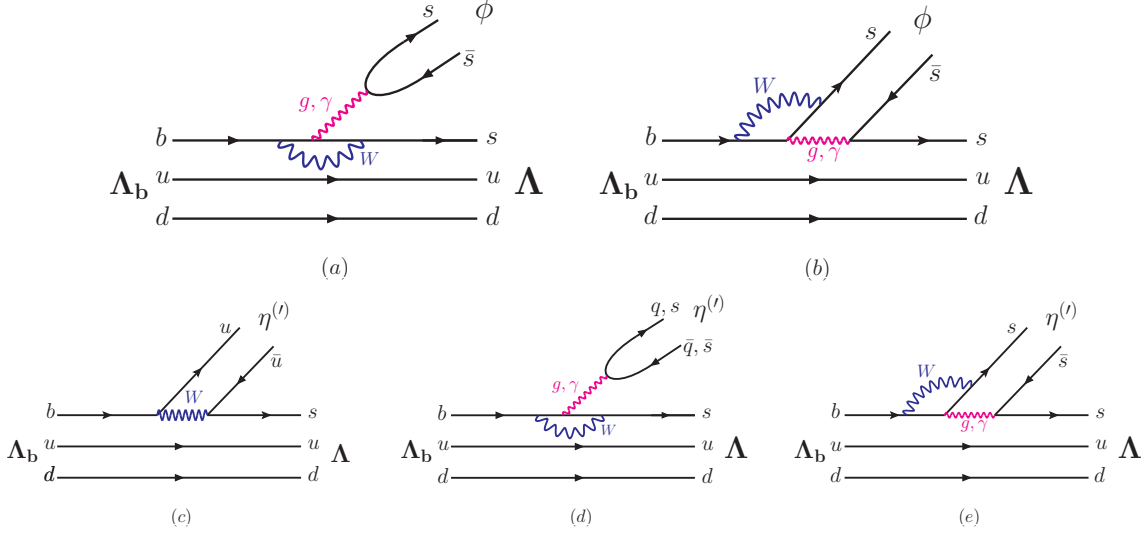


FIG. 1. Feynman diagrams (a,b) and (c,d,e) from $\Lambda_b^0 \rightarrow \Lambda \phi$ and $\Lambda_b \rightarrow \Lambda \eta^{(\prime)}$ decays, respectively.

II. FORMALISM

In terms of the effective Hamiltonian for the charmless $b \rightarrow ss\bar{s}$ transition at the quark level shown Fig. 1, the amplitude of $\Lambda_b \rightarrow \Lambda \phi$ based on the factorization approach can be derived as [17]

$$\mathcal{A}(\Lambda_b \rightarrow \Lambda \phi) = \frac{G_F}{\sqrt{2}} \alpha_3 \langle \phi | \bar{s} \gamma_\mu s | 0 \rangle \langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle, \quad (3)$$

with G_F the Fermi constant, $V_{q_1 q_2}$ the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $\alpha_3 = -V_{tb} V_{ts}^* (a_3 + a_4 + a_5 - a_9/2)$, where $a_i \equiv c_i^{eff} + c_{i\pm 1}^{eff}/N_c$ for $i = \text{odd}$ (even) are composed of the effective Wilson coefficients c_i^{eff} defined in Ref. [17] with the color number N_c . As depicted in Fig. 1, the amplitudes of $\Lambda_b \rightarrow \Lambda \eta^{(\prime)}$ are given by

$$\begin{aligned} \mathcal{A}(\Lambda_b \rightarrow \Lambda \eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} \Bigg\{ & \left[\beta_2 \langle \eta^{(\prime)} | \bar{q} \gamma_\mu \gamma_5 q | 0 \rangle + \beta_3 \langle \eta^{(\prime)} | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle \right] \langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle \\ & + \beta_6 \langle \eta^{(\prime)} | \bar{s} \gamma_5 s | 0 \rangle \langle \Lambda | \bar{s} (1 - \gamma_5) b | \Lambda_b \rangle \Bigg\}, \end{aligned} \quad (4)$$

with $q = u$ or d , where $\beta_2 = -V_{ub} V_{us}^* a_2 + V_{tb} V_{ts}^* (2a_3 - 2a_5 + a_9/2)$, $\beta_3 = V_{tb} V_{ts}^* (a_3 + a_4 - a_5 - a_9/2)$, and $\beta_6 = V_{tb} V_{ts}^* 2a_6$. The matrix elements of the $\Lambda_b \rightarrow \Lambda$ baryon transition in Eqs. (3) and (4) have been parameterized as [18]

$$\begin{aligned} \langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle &= \bar{u}_\Lambda (f_1 \gamma_\mu - g_1 \gamma_\mu \gamma_5) u_{\Lambda_b}, \\ \langle \Lambda | \bar{s} (1 - \gamma_5) b | \Lambda_b \rangle &= \bar{u}_\Lambda (f_S \gamma_\mu - g_P \gamma_\mu \gamma_5) u_{\Lambda_b}, \end{aligned} \quad (5)$$

where f_1 , g_1 , f_S , and g_P are the form factors, with $f_S = [(m_{\Lambda_b} - m_\Lambda)/(m_b - m_s)]f_1$ and $g_P = [(m_{\Lambda_b} + m_\Lambda)/(m_b + m_s)]g_1$ by virtue of equations of motion. Note that, in Eq. (5), we have neglected the form factors related to $\bar{u}_\Lambda \sigma_{\mu\nu} q^\nu (\gamma_5) u_{\Lambda_b}$ and $\bar{u}_\Lambda q_\mu (\gamma_5) u_{\Lambda_b}$ that flip the helicity [19]. With the double-pole momentum dependences, f_1 and g_1 can be written as [8]

$$f_1(q^2) = \frac{f_1(0)}{(1 - q^2/m_{\Lambda_b}^2)^2}, \quad g_1(q^2) = \frac{g_1(0)}{(1 - q^2/m_{\Lambda_b}^2)^2}, \quad (6)$$

where we have taken $C_F(\Lambda_b \rightarrow \Lambda) \equiv f_1(0) = g_1(0)$ as the leading approximation based on the $SU(3)$ flavor and $SU(2)$ spin symmetries [20, 21]. We remark that the perturbative corrections to the $\Lambda_b \rightarrow \Lambda$ transition form factors from QCD sum rules have been recently computed in Ref. [22]. Clearly, for more precise evaluations of the form factors, these corrections should be included.

The matrix elements in Eqs. (3) and (4) for the meson productions read [23]

$$\begin{aligned} \langle \phi | \bar{s} \gamma_\mu s | 0 \rangle &= m_\phi f_\phi \varepsilon_\mu^*, \quad \langle \eta^{(\prime)} | \bar{q} \gamma_\mu \gamma_5 q | 0 \rangle = -\frac{i}{\sqrt{2}} f_{\eta^{(\prime)}}^q q_\mu, \\ \langle \eta^{(\prime)} | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle &= -i f_{\eta^{(\prime)}}^s q_\mu, \quad 2m_s \langle \eta^{(\prime)} | \bar{s} \gamma_5 s | 0 \rangle = -i h_{\eta^{(\prime)}}^s, \end{aligned} \quad (7)$$

with the polarization ε_μ^* and four-momentum q_μ vectors for ϕ and $\eta^{(\prime)}$, respectively, where f_ϕ , $f_{\eta^{(\prime)}}^q$, and $h_{\eta^{(\prime)}}^s$ are decay constants. Unlike the usual decay constants, $f_{\eta^{(\prime)}}^q$ and $f_{\eta^{(\prime)}}^s$ are the consequences of the $\eta - \eta'$ mixing, in which the Feldmann, Kroll and Stech (FKS) scheme is adopted as [24]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (8)$$

with $|\eta_q\rangle = (|u\bar{u} + d\bar{d}\rangle)/\sqrt{2}$ and $|\eta_s\rangle = |s\bar{s}\rangle$, where the mixing angle is extracted as $\phi = (39.3 \pm 1.0)^\circ$. As a result, $f_{\eta^{(\prime)}}^q$ and $f_{\eta^{(\prime)}}^s$ actually mix with the decay constants f_q and f_s for η_q and η_s , respectively. Note that $h_{\eta^{(\prime)}}^s$ in Eq. (7) contains the contribution from the QCD anomaly, given by

$$2m_s \langle \eta^{(\prime)} | \bar{s} i \gamma_5 s | 0 \rangle = \partial^\mu \langle \eta^{(\prime)} | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle + \langle \eta^{(\prime)} | \frac{\alpha_s}{4\pi} G \tilde{G} | 0 \rangle, \quad (9)$$

where α_s is the strong coupling constant, $G(\tilde{G})$ is the (dual) gluon field tensor, $\partial^\mu \langle \eta^{(\prime)} | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle = f_{\eta^{(\prime)}} m_{\eta^{(\prime)}}^2$, and $\langle \eta^{(\prime)} | \alpha_s G \tilde{G} | 0 \rangle \equiv 4\pi a_{\eta^{(\prime)}}$. Explicitly, one has [23]

$$h_{\eta^{(\prime)}}^s = a_{\eta^{(\prime)}} + f_{\eta^{(\prime)}}^s m_{\eta^{(\prime)}}^2, \quad (10)$$

which will be used in the numerical analysis.

TABLE I. α_i (β_i) with $N_c = 2, 3$, and ∞ .

α_i (β_i)	$N_c = 2$	$N_c = 3$	$N_c = \infty$
$10^4 \alpha_3$	$-21.97 - 4.47i$	$-15.51 - 3.39i$	$-2.59 - 1.24i$
$10^4 \beta_2$	$-11.93 + 1.71i$	$-9.42 + 0.23i$	$-4.41 - 2.73i$
$10^4 \beta_3$	$7.58 + 3.18i$	$10.07 + 3.39i$	$15.05 + 3.82i$
$10^4 \beta_6$	$47.48 + 6.44i$	$49.55 + 6.87i$	$53.71 + 7.73i$

III. NUMERICAL RESULTS AND DISCUSSIONS

For our numerical analysis, the CKM matrix elements in the Wolfenstein parameterization are given by [25]

$$(V_{ub}, V_{us}, V_{tb}, V_{ts}) = (A\lambda^3(\rho - i\eta), \lambda, 1, -A\lambda^2), \quad (11)$$

with $(\lambda, A, \rho, \eta) = (0.225, 0.814, 0.120 \pm 0.022, 0.362 \pm 0.013)$. In Table I, we fix $N_c = 3$ for a_i but shift it from 2 to ∞ in the generalized version of the factorization approach to take into account the non-factorizable effects as the uncertainty. For the form factors, we use $C_F(\Lambda_b \rightarrow \Lambda) = -\sqrt{2/3} C_F(\Lambda_b \rightarrow p)$ [21] with $C_F(\Lambda_b \rightarrow p) = 0.136 \pm 0.009$ [8]. Apart from $f_\phi = 0.231$ GeV [26], we adopt the decay constants for η and η' from Ref. [23], given by

$$\begin{aligned} (f_\eta^q, f_{\eta'}^q, f_\eta^s, f_{\eta'}^s) &= (0.108, 0.089, -0.111, 0.136) \text{ GeV}, \\ (h_\eta^s, h_{\eta'}^s) &= (-0.055, 0.068) \text{ GeV}^3, \end{aligned} \quad (12)$$

respectively. Subsequently, we obtain the branching ratios, given in Table II.

As seen in Table I, α_3 for $\Lambda_b \rightarrow \Lambda\phi$ is sensitive to the non-factorizable effects. In comparison with the data in Table II and Eq. (1), the $\Lambda_b \rightarrow \Lambda\phi$ decay is judged to receive the non-factorizable effects with $N_c = 2$, such that $\mathcal{B}(\Lambda_b \rightarrow \Lambda\phi) = (3.53 \pm 0.24) \times 10^{-6}$. With $\mathcal{B}(\phi \rightarrow K^+ K^-) = (48.5 \pm 0.5)\%$ [25], we get $\mathcal{B}(\Lambda_b \rightarrow \Lambda(\phi \rightarrow K^+ K^-)) = (1.71 \pm 0.12) \times 10^{-6}$, which is much lower than the data of $(15.9 \pm 4.4) \times 10^{-6}$ in Eq. (2), leaving some room for other contributions, such as the resonant $\Lambda_b \rightarrow K^- N^*$, $N^* \rightarrow \Lambda K^+$ decay with N^* denoted as the higher-wave baryon state. Here, we would suggest a more accurate experimental examination on the ΛK invariant mass spectrum, which depends on the peak around the threshold of $m_{\Lambda K} \simeq m_\Lambda + m_K$, while the Dalitz plot might possibly reveal the signal [13].

The result of $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta) = (1.47 \pm 0.35) \times 10^{-6}$ in Table II shows a consistent result with the data due to its large uncertainty. On the other hand, $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta') = (1.83 \pm 0.58) \times 10^{-6}$ agrees with the experimental upper bound. The η and η' modes with $\mathcal{B} \simeq 10^{-6}$ are mainly resulted from the form factor $C_F(\Lambda_b \rightarrow \Lambda) \sim 0.14$ extracted in Ref. [8] in agreement with the calculation in QCD models [6, 18, 19], which explains why $\mathcal{B}(\Lambda_b \rightarrow pK^-, p\pi^-)$ are also around 10^{-6} [25]. As seen in Table II, our results for the $\eta^{(\prime)}$ modes are smaller than those in Ref. [15]. In addition, we note that, the result of $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta') = 11.33(3.24) \times 10^{-6}$ in the QCD sum rule (pole) model [15], apparently exceeds the data. However, the predictions for $\Lambda_b \rightarrow \Lambda\eta$ in Ref. [15] are still consistent with the current data. We also point out that the relation of $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta) \simeq \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta')$ still holds as in Ref. [15].

It is known that the gluon content of $\eta^{(\prime)}$ can contribute to the flavor-singlet $B \rightarrow K\eta^{(\prime)}$ decays in three ways [23]: (i) the $b \rightarrow sgg$ amplitude related to the effective charm decay constant, (ii) the spectator scattering involving two gluons, and (iii) the singlet weak annihilation. It is interesting to ask if these three production mechanisms are also relevant to the corresponding $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$ decays. For (i), its contribution to $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$ has been demonstrated to be small [15] since, by effectively relating $b \rightarrow sgg$ to $b \rightarrow sc\bar{c}$, the $c\bar{c}$ vacuum annihilation of $\eta^{(\prime)}$ is suppressed due to the decay constants $(f_\eta^c, f_{\eta'}^c) \simeq (-1, -3)$ MeV [23] being much smaller than $f_{\eta^{(\prime)}}^q$ in Eq. (12). For (ii), since one of the gluons from the spectator quark connects to the recoiled $\eta^{(\prime)}$, the contribution belongs to the non-factorizable effect, which has been inserted into the effective number of N_c (from 2 to ∞) in our generalized factorization approach. For (iii), it is the sub-leading power contribution which does not

TABLE II. Numerical results for the branching ratios with the first and second errors from the non-factorizable effects and the form factors, respectively, in comparison with the experimental data [2, 3] and the study in Ref. [15]. Note that, in column 3, the two values without and with the parenthesis correspond to the form factors in the approach of QCD sum rules and the pole model, respectively.

decay mode	our results	data [2, 3]	Ref. [15]
$10^6 \mathcal{B} \Lambda_b \rightarrow \Lambda\phi$	$1.77_{-1.71}^{+1.76} \pm 0.24$	5.18 ± 1.29	...
$10^6 \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta)$	$1.47_{-0.13}^{+0.29} \pm 0.20$	$9.3_{-5.3}^{+7.3}$	11.47 (2.95)
$10^6 \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta')$	$1.83_{-0.18}^{+0.55} \pm 0.25$	< 3.1	11.33 (3.24)

contribute to $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$.

Finally, we remark that, in the b -hadron decays, such as those of B and Λ_b , the generalized factorization with the floating $N_c = 2 \rightarrow \infty$ [17] can empirically estimate the non-factorizable effects, such that it can be used to explain the data as well as make predictions. On the other hand, the QCD factorization [23] could in general calculate the non-factorizable effects in some specific processes. Although the current existing studies on $\Lambda_b \rightarrow \Lambda(\phi, \eta^{(\prime)})$ are based on the generalized factorization, it is useful to calculate these decay modes in the QCD factorization. In particular, since the decay of $\Lambda_b \rightarrow \Lambda\phi$ with $N_c = 2$ has shown to be sensitive to the non-factorizable effects, its study in the QCD factorization is clearly interesting.

IV. CONCLUSIONS

In sum, we have studied the charmless two-body $\Lambda_b \rightarrow \Lambda\phi$ and $\Lambda_b \rightarrow \Lambda\eta^{(\prime)}$ and three-body $\Lambda_b \rightarrow \Lambda K^+ K^-$ decays. By predicting $\mathcal{B}(\Lambda_b \rightarrow \Lambda\phi) = (3.53 \pm 0.24) \times 10^{-6}$ to agree with the observation, we have found that $\mathcal{B}(\Lambda_b \rightarrow \Lambda(\phi \rightarrow) K^+ K^-) = (1.71 \pm 0.12) \times 10^{-6}$ cannot explain the observed $\mathcal{B}(\Lambda_b \rightarrow \Lambda K^+ K^-) = (15.9 \pm 4.4) \times 10^{-6}$, which leaves much room for the contribution from the resonant $\Lambda_b \rightarrow K^- N^*$, $N^* \rightarrow \Lambda K^+$ decay. We have obtained $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta, \Lambda\eta') = (1.47 \pm 0.35, 1.83 \pm 0.58) \times 10^{-6}$ in comparison with the data of $(9.3_{-5.3}^{+7.3}, < 3.1) \times 10^{-6}$, respectively. In addition, our results still support the relation of $\mathcal{B}(\Lambda_b \rightarrow \Lambda\eta) \simeq \mathcal{B}(\Lambda_b \rightarrow \Lambda\eta')$. It is clear that future more precise experimental measurements on the present Λ_b decays are important to test the QCD models, in particular the generalized factorization one.

ACKNOWLEDGMENTS

The work was supported in part by National Center for Theoretical Sciences, National Science Council (NSC-101-2112-M-007-006-MY3) and MoST (MoST-104-2112-M-007-003-MY3).

[1] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **103**, 031801 (2009).

- [2] R. Aaij *et al.* [LHCb Collaboration], arXiv:1603.02870 [hep-ex].
- [3] R. Aaij *et al.* [LHCb Collaboration], JHEP **1509**, 006 (2015).
- [4] C.D. Lu, Y.M. Wang, H. Zou, A. Ali and G. Kramer, Phys. Rev. D **80**, 034011 (2009).
- [5] S. Wang, J. Huang and G. Li, Chin. Phys. C **37**, 063103 (2013).
- [6] Z.T. Wei, H.W. Ke and X.Q. Li, Phys. Rev. D **80**, 094016 (2009).
- [7] J. Zhu, H.W. Ke and Z.T. Wei, arXiv:1603.02800 [hep-ph].
- [8] Y.K. Hsiao and C.Q. Geng, Phys. Rev. D **91**, 116007 (2015); PoS FPCP **2015**, 073 (2015).
- [9] Y. Liu, X.H. Guo and C. Wang, Phys. Rev. D **91**, 016006 (2015).
- [10] X.H. Guo and A.W. Thomas, Phys. Rev. D **58**, 096013 (1998).
- [11] S. Arunagiri and C.Q. Geng, Phys. Rev. D **69**, 017901 (2004).
- [12] O. Leitner, Z.J. Ajaltouni and E. Conte, hep-ph/0602043.
- [13] R. Aaij *et al.* [LHCb Collaboration], arXiv:1603.00413 [hep-ex].
- [14] C.H. Chen and C.Q. Geng, Phys. Rev. D **63**, 054005 (2001); Phys. Rev. D **63**, 114024 (2001); Phys. Rev. D **64**, 074001 (2001); Phys. Rev. D **65**, 091502 (2002).
- [15] M.R. Ahmady, C.S. Kim, S. Oh and C. Yu, Phys. Lett. B **598**, 203 (2004).
- [16] T. Gutsche, M.A. Ivanov, J.G. Krner, V.E. Lyubovitskij and P. Santorelli, Phys. Rev. D **90**, 114033 (2014).
- [17] A. Ali, G. Kramer and C.D. Lu, Phys. Rev. D **58**, 094009 (1998).
- [18] A. Khodjamirian, C. Klein, T. Mannel and Y.M. Wang, JHEP **1109**, 106 (2011); T. Mannel and Y.M. Wang, JHEP **1112**, 067 (2011).
- [19] T. Gutsche, M.A. Ivanov, J.G. Krner, V.E. Lyubovitskij and P. Santorelli, Phys. Rev. D **88**, 114018 (2013).
- [20] G.P. Lepage and S.J. Brodsky, Phys. Rev. Lett. **43**, 545(1979) [Erratum-ibid. **43**, 1625 (1979)].
- [21] Y.K. Hsiao, P.Y. Lin, C.C. Lih and C.Q. Geng, Phys. Rev. D **92**, 114013 (2015).
- [22] Y.M. Wang and Y.L. Shen, JHEP **1602**, 179 (2016).
- [23] M. Beneke and M. Neubert, Nucl. Phys. B **651**, 225 (2003).
- [24] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999).
- [25] K.A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).
- [26] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014029 (2005).